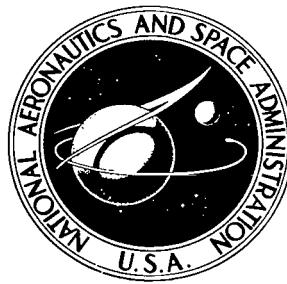


NASA TECHNICAL NOTE



NASA TN D-4290

C.1

NASA TN D-4290

LOAN COPY: RETU  
AFWL (WLIL-  
KIRTLAND AFB, N



TECH LIBRARY KAFB, NM

# A LOGARITHMIC ENCODER FOR BINARY WORD COMPRESSION

by Joseph A. Sciulli

Goddard Space Flight Center  
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MARCH 1968

**TECH LIBRARY KAFB, NM**



**0130906**

## **A LOGARITHMIC ENCODER FOR BINARY WORD COMPRESSION**

**By Joseph A. Sciulli**

**Goddard Space Flight Center  
Greenbelt, Md.**

**NATIONAL AERONAUTICS AND SPACE ADMINISTRATION**

---

**For sale by the Clearinghouse for Federal Scientific and Technical Information  
Springfield, Virginia 22151 – CFSTI price \$3.00**

## ABSTRACT

During the past several years efficient information transmission and processing techniques have attracted wide interest and have increased applicability. The present paper describes a logarithmic encoding device which has had particular application in energetic particle detection experiments. The paper provides a generalized encoding error analysis in order to evaluate the performance of the device. Both peak and average error are derived in terms of word size and desired accuracy. The implementation of a flexible logarithmic encoder is also described.

## CONTENTS

Abstract . . . . .	ii
INTRODUCTION . . . . .	1
ENCODING PROCEDURE . . . . .	1
ENCODING ERROR . . . . .	2
IMPLEMENTATION . . . . .	5
CONCLUSIONS . . . . .	5
References . . . . .	6
Appendix—Listing of $\bar{E}_F$ and $E_{F[\max]}$ for Combinations of n and k . . . . .	7

# A LOGARITHMIC ENCODER FOR BINARY WORD COMPRESSION

by

Joseph A. Sciulli

Goddard Space Flight Center

## INTRODUCTION

Growing interest in the development of efficient information transmission techniques is stimulating much theoretical and experimental work. Particular emphasis has been placed on the development of video data compression techniques which exploit the predictability (or redundancy) of the data (Reference 1). Many of these techniques are not specifically designed for the video information source and can, in principle, be applied to any information source. For example, recent experiments have applied a simple predictor/encoder compression technique to energetic particle experiment data. The experimental data were obtained from a multi-channel device flown on the 1963-38C-APL satellite in which each channel was designed to detect particles in a specific energy region. Typically, an experiment of this kind requires a rather large word size because of the wide range of possible sample values. (The experiment considered used 16 bits per sample.) The compression simulations showed that average energy savings of approximately 4 db can be achieved with an allowable peak error of approximately 5 percent.

The main difficulty with such an approach, however, is defining an error criterion which is suitable over the entire range of possible sample values. Logarithmic encoding, however, is extremely useful in experiments of this type which involve wide dynamic range. The function of the encoder is to reduce the number of transmitted bits per sample while maintaining a relatively small error in the word reconstructed at the receiver. This paper describes a logarithmic encoding procedure and develops an expression for the error between the original sample value and the sample value reconstructed at the receiver. In addition, several design considerations are outlined and the operation of the device is discussed briefly. This procedure results in a fixed compression, independent of the *predictability* of the data. Since the bit rate at the output of the encoder is constant, there is no need to consider the yet-unsolved buffer queuing problem.

## ENCODING PROCEDURE

Suppose each sample is originally represented by an n-bit binary word  $S$  with components  $(\sigma_{n-1}, \sigma_{n-2}, \dots, \sigma_{n-j}, \sigma_{n-j-1} \dots \sigma_0)$ . The most significant bit (MSB) is  $\sigma_{n-1}$ ;  $\sigma_0$  is the least

significant bit (LSB); and  $\sigma_{n-j}$  is the first component (from the left) which is a 1. We now describe an encoder which converts  $S$  into two subwords, the characteristic ( $C$ ) and the mantissa ( $M$ ). The mantissa is a  $k$ -bit word specifying the  $k$  MSBs following  $\sigma_{n-j}$ . The position of  $\sigma_{n-j-k+1}$  is specified by  $C$ , an  $r$ -bit word. The encoder output, therefore, consists of  $(r + k)$  bits where  $(r + k) \leq n$ . The operation is best illustrated by an example:

let  $n = 19$ ,  $k = 4$ , and  $r = 4$  and suppose the input word is  $S = 1011001110110010110$ .

Here the first 1 appears at the MSB, hence  $j = 1$ . The 4 bits of  $M$  must then consist of the second, third, fourth, and fifth MSBs of  $S$  (i.e.,  $M = 0110$ ). The position of the LSB of  $M$  must then be specified by  $C$ . Thus,  $C = n - k - j + 1 = 15$  or  $C = 1111$ . The encoder output is then,

$$\underbrace{1111}_{C} \quad \underbrace{0110}_{M} .$$

Some additional examples are

<u>Input</u>	<u>Output</u>
11111111111111111111	11111111
10000000000000000000	11110000
0000000000110000000	01011000
00000000000000010000	00010000

This example was given by Schaefer (Reference 2).

The general design problem consists of choosing  $k$  and  $r$  for some specified  $n$ . Since  $k$  determines how closely the encoded word approximates the input word ( $n$ -bits),  $k$  should be chosen to satisfy the error specification. After  $k$  is fixed,  $r$  can be chosen to minimize the number of bits in the output word. The bit compression ratio  $n/(r + k)$  then provides a useful system performance measure. In the next section error expressions assuming transmission in a noiseless channel are developed.

## ENCODING ERROR

In practice, an experimenter might specify an upper limit on either the average error, the maximum error, or both. Here both the average error and the maximum error for the special case of equally likely samples are calculated.

The input word  $S$  for some value of  $j$  is, by definition,

$$S = \sum_{i=0}^{n-j} \sigma_i 2^i . \quad (1)$$

According to the encoding rule, the word reconstructed at the receiver must be

$$\tilde{S} = \sum_{i=n-j-k}^{n-j} \sigma_i 2^i , \quad (2)$$

and the error for a given  $j$

$$E_j = S - \tilde{S} = \sum_{i=0}^{n-j-k-1} \sigma_i 2^i . \quad (3)$$

We may now calculate the expected error for a given  $j$  according to

$$\bar{E}_j = E\{E_j\} = \sum_{i=0}^{n-j-k-1} E(\sigma_i) 2^i . \quad (4)$$

If all input words are equally likely,  $E(\sigma_i) = 1/2$ . Therefore,

$$\bar{E}_j = \frac{1}{2} \sum_{i=0}^{n-j-k-1} 2^i = \frac{1}{2} [2^{n-j-k} - 1] . \quad (5)$$

To determine the average error we must average  $\bar{E}_j$  over all values of  $j$  for which an error can occur. That is,

$$\bar{E} = \sum_j \bar{E}_j p(j) ; \quad p(j) = \begin{cases} \frac{1}{2^j} & \text{for } j = 1, 2, \dots, (n-k-1) \\ \frac{1}{2^{n-k-1}} & \text{for } j = (n-k) \end{cases}$$

where  $\bar{E}_j = 0$  for  $j = (n-k)$ . Therefore,

$$\begin{aligned} \bar{E} &= \sum_{j=1}^{n-k-1} \frac{1}{2} [2^{n-j-k} - 1] \frac{1}{2^j} \\ &= 2^{n-k-1} \left\{ \sum_{j=1}^{n-k-1} 2^{-2j} \right\} - \frac{1}{2} \left\{ \sum_{j=1}^{n-k-1} 2^{-j} \right\} . \end{aligned} \quad (6)$$

Since each term in braces is a geometric series,

$$\bar{E} = 2^{n-k-1} \left\{ \frac{1}{3} (1 - 2^{-2(n-k-1)}) \right\} - \frac{1}{2} \left\{ 1 - 2^{-(n-k-1)} \right\}, \quad (7)$$

and, collecting terms,

$$\bar{E} = \frac{1}{3} \left[ 2^{n-k-1} + 2^{-(n-k)} - \frac{3}{2} \right]. \quad (8)$$

We can also calculate the maximum error. Returning to Equation 3, the error for a given  $j$  is

$$E_j = \sum_{i=0}^{n-j-k-1} \sigma_i 2^i.$$

Now the maximum error must obviously occur for  $j = 1$  and  $\sigma_i = 1$  for all  $i$ . Therefore,

$$E_{\max} = \max_j E_j = \sum_{i=0}^{n-k-2} 2^i = [2^{n-k-1} - 1]. \quad (9)$$

The average error as a fraction of full scale is

$$\bar{E}_F = \frac{\left[ 2^{n-k-1} + 2^{-(n-k)} - \frac{3}{2} \right]}{3(2^n - 1)}, \quad (10)$$

and, for  $(n - k) \gg 1$ ,

$$\bar{E}_F \doteq \frac{2^{n-k-1}}{3(2^n)} = \frac{1}{3} \cdot \frac{1}{2^{k+1}}. \quad (11)$$

Similarly, the maximum error as a fraction of full scale is

$$E_{F[\max]} = \frac{2^{n-k-1} - 1}{2^n - 1}, \quad (12)$$

and, again, for  $(n - k) \gg 1$ ,

$$E_{F[\max]} \doteq \frac{1}{2^{k+1}}. \quad (13)$$

The appendix lists both  $\bar{E}_F$  and  $E_{F[\max]}$  for various combinations of  $n$  and  $k$ . Thus, for a given  $n$ , a  $k$  can be chosen, according to Equations 10 and 12, to satisfy given error requirements. After the value of  $k$  is established  $r$  should be chosen as the smallest integer satisfying

$$(n - k) \leq 2^r - 1 ,$$

or

$$r \geq \log_2 (n - k + 1) . \quad (14)$$

If equality is obtained in Equation 14, then all possible values of C can occur.

## IMPLEMENTATION

The implementation of this device is quite simple. A generalized system is shown in Figure 1. The n-bit input word is presented serially (MSB first) to a k-bit register. The input word is shifted until a 1 appears in the kth bit of the register. The next clock pulse changes the state of the control flip flop, inhibiting the shift register and starting the r-bit counter. At the nth clock pulse the shift register contains the k bits of M and the counter contains the r bits of C. The contents of the register and counter can then be transferred to an output register. The elements of the encoder are then reset, and the device is prepared to receive the next input word.

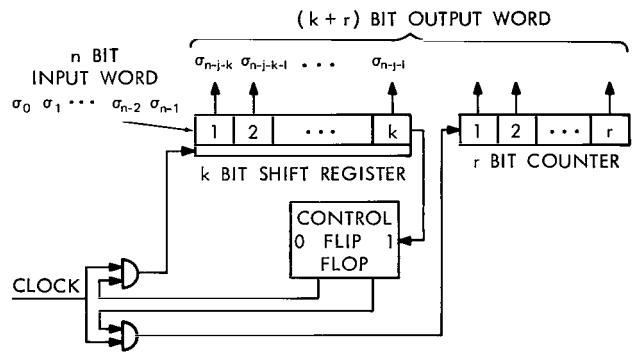


Figure 1—Implementation of the encoder.

## CONCLUSIONS

This encoding procedure provides a useful device which achieves fixed, but modest, bit compression. For example, the 16-bit words of the energetic particle experiment could be encoded to 8 bits ( $k = 4$ ,  $r = 4$ ), resulting in a bit compression ratio of 2:1. This gives an average error of approximately 1 percent and a maximum error of about 3 percent. Moreover, this device could be combined with a zero-order predictor/run-length encoder to achieve further compression. The zero order hold compression might even be applied in the bit planes of C since these should be relatively quiescent from sample to sample.

For a slight increase in complexity, the logarithmic encoder could be made more flexible by varying  $k$  and  $r$  on command from the ground. Thus, the experimenter would have the capability of selecting the allowable error in the data depending on the activity of his experiment at a given time.

Goddard Space Flight Center  
National Aeronautics and Space Administration  
Greenbelt, Maryland, August 1, 1967  
160-44-05-14-51

## REFERENCES

1. Kutz, R. L. and Sciulli, J. A., "An Adaptive Image Data Compression System and its Performances in a Noisy Channel," presented at the International Symposium on Information Theory, San Ramo, Italy, September 11-15, 1967, submitted to IEEE Transactions on Information Theory.
2. Schaefer, D. H., "Logarithmic Compression of Binary Numbers," *Proceedings of the IRE* (Correspondence) 49 (7): July 1961.

Appendix

**Listing of  $\bar{E}_F$  and  $E_F[\max]$  for Combinations of n and k**

n	k	$\bar{E}_F$	$E_F[\max]$	n	k	$\bar{E}_F$	$E_F[\max]$
2	1	0.	0.	12	1	0.09323162	0.24981684
3	1	0.03571428	0.14285714	12	2	0.04155482	0.12478632
3	2	0.	0.	12	3	0.02071647	0.06227105
4	1	0.05833333	0.20000000	12	4	0.01029743	0.03101343
4	2	0.01666667	0.06666665	12	5	0.00508814	0.01538461
4	3	0.	0.	12	6	0.00248397	0.00757021
5	1	0.07056452	0.22580644	12	7	0.00118284	0.00366300
5	2	0.02827580	0.09677419	12	8	0.00053418	0.00170940
5	3	0.00806452	0.03225806	12	9	0.00021368	0.00073260
5	4	0.	0.	12	10	0.00006105	0.00024420
6	1	0.07688492	0.23809523	12	11	0.	0.
6	2	0.03472222	0.11111110	13	1	0.08328247	0.24990843
6	3	0.01388889	0.04761904	13	2	0.04161073	0.12489317
6	4	0.00396825	0.01587301	13	3	0.02077487	0.06238554
6	5	0.	0.	13	4	0.01035697	0.03113173
7	1	0.08009350	0.24409448	13	5	0.00514808	0.01550482
7	2	0.03813976	0.11811023	13	6	0.00254375	0.00769137
7	3	0.01722441	0.05511811	13	7	0.00124183	0.00378464
7	4	0.00688976	0.02362204	13	8	0.00059135	0.00183128
7	5	0.00196850	0.00787401	13	9	0.00026706	0.00085450
7	6	0.	0.	13	10	0.00010682	0.00036626
8	1	0.08170956	0.24705882	13	11	0.00003052	0.00012209
8	2	0.03988970	0.12156862	13	12	0.	0.
8	3	0.01999509	0.05882353	14	1	0.08330790	0.24995422
8	4	0.00857843	0.02745098	14	2	0.04163869	0.12494659
8	5	0.00343137	0.01176471	14	3	0.02080409	0.06244277
8	6	0.00098039	0.00392157	14	4	0.01038680	0.03119086
8	7	0.	0.	14	5	0.00517817	0.01556491
9	1	0.08252048	0.24853229	14	6	0.00257389	0.00775193
9	2	0.04077482	0.12328767	14	7	0.00127180	0.00384545
9	3	0.01990582	0.06066536	14	8	0.00062088	0.00189220
9	4	0.00947896	0.02935421	14	9	0.00029565	0.00091558
9	5	0.00428082	0.01369863	14	10	0.00013352	0.00042727
9	6	0.00171232	0.00587084	14	11	0.00005341	0.00018311
9	7	0.00048923	0.00195695	14	12	0.00001526	0.00006104
9	8	0.	0.	14	13	0.	0.
10	1	0.08292667	0.24926686	15	1	0.08332061	0.24997710
10	2	0.04121991	0.12414467	15	2	0.04165268	0.12497329
10	3	0.02036748	0.06158357	15	3	0.02081871	0.06247138
10	4	0.00994318	0.03030302	15	4	0.01040173	0.03122043
10	5	0.00473484	0.01466275	15	5	0.00519324	0.01559495
10	6	0.00213832	0.00684261	15	6	0.00258900	0.00778221
10	7	0.00085533	0.00293255	15	7	0.00128690	0.00387584
10	8	0.00024438	0.00097752	15	8	0.00063588	0.00192266
10	9	0.	0.	15	9	0.00031043	0.00094607
11	1	0.08312994	0.24963360	15	10	0.00014782	0.00045777
11	2	0.04144308	0.12457254	15	11	0.00006676	0.00021362
11	3	0.02059989	0.06204201	15	12	0.00002670	0.00009155
11	4	0.01017877	0.03077674	15	13	0.00000763	0.00003052
11	5	0.00496916	0.01514411	15	14	0.	0.
11	6	0.00236627	0.00732780				
11	7	0.00106864	0.00341964				
11	8	0.00042745	0.00146556				
11	9	0.00012213	0.00048857				
11	10	0.	0.				

n	k	$\bar{E}_F$	$E_F [max]$	n	k	$\bar{E}_F$	$E_F [max]$
16	1	0.08332697	0.24998855	19	1	0.08333253	0.24999856
16	2	0.04165967	0.12498664	19	2	0.04166579	0.12499832
16	3	0.02082602	0.06248559	19	3	0.02083241	0.06249820
16	4	0.01040919	0.03123521	19	4	0.01041573	0.03124814
16	5	0.00520078	0.01560997	19	5	0.00520738	0.01562312
16	6	0.00259658	0.00779735	19	6	0.00260322	0.00781060
16	7	0.00129448	0.00389104	19	7	0.00130113	0.00390434
16	8	0.00064344	0.00193789	19	8	0.00065009	0.00195122
16	9	0.00031793	0.00096131	19	9	0.00032456	0.00097456
16	10	0.00015521	0.00047302	19	10	0.00016180	0.00048637
16	11	0.00007391	0.00022888	19	11	0.00008043	0.00024223
16	12	0.00003338	0.00010681	19	12	0.00003974	0.00012016
16	13	0.00001335	0.00004578	19	13	0.00001940	0.00005913
16	14	0.00000381	0.00001526	19	14	0.00000924	0.00002361
16	15	0.	0.	19	15	0.00000417	0.00001335
17	1	0.08333015	0.24999427	19	16	0.00000157	0.00000572
17	2	0.04156316	0.12499332	19	17	0.00000048	0.00000191
17	3	0.02082967	0.06249284	19	18	0.	0.
17	4	0.01041293	0.03124260	20	1	0.08333293	0.24999928
17	5	0.00520456	0.01561748	20	2	0.04166622	0.12499916
17	6	0.00260037	0.00780492	20	3	0.02083287	0.06249910
17	7	0.00129828	0.00389854	20	4	0.01041619	0.03124907
17	8	0.00064723	0.00194550	20	5	0.00520786	0.01562405
17	9	0.00032172	0.00096893	20	6	0.00260369	0.00781155
17	10	0.00015897	0.00048055	20	7	0.00130160	0.00390530
17	11	0.00007761	0.00023651	20	8	0.00065056	0.00195217
17	12	0.00003695	0.00011444	20	9	0.00032504	0.00097561
17	13	0.00001669	0.00005341	20	10	0.00016228	0.00048733
17	14	0.00000668	0.00002289	20	11	0.00008090	0.00024319
17	15	0.00000191	0.00000753	20	12	0.00004021	0.00012112
17	16	0.	0.	20	13	0.00001987	0.00006008
18	1	0.08333174	0.24999713	20	14	0.00000970	0.00002955
18	2	0.04166491	0.12499665	20	15	0.00000462	0.00001431
18	3	0.02083150	0.06249642	20	16	0.00000209	0.00000658
18	4	0.01041479	0.03124630	20	17	0.00000083	0.00000286
18	5	0.00520644	0.01562124	20	18	0.00000024	0.00000095
18	6	0.00260276	0.00780871	20	19	0.	0.
18	7	0.00130018	0.00390244				
18	8	0.00064913	0.00194931				
18	9	0.00032362	0.00097275				
18	10	0.00016086	0.00048447				
18	11	0.00007948	0.00024033				
18	12	0.00003880	0.00011826				
18	13	0.00001848	0.00005722				
18	14	0.00000834	0.00002670				
18	15	0.00000334	0.00001144				
18	16	0.00000095	0.00000381				
18	17	0.	0.				

National Aeronautics and Space Administration  
WASHINGTON, D. C.  
OFFICIAL BUSINESS

FIRST CLASS MAIL

POSTAGE AND FEES PAID  
NATIONAL AERONAUTICS AND  
SPACE ADMINISTRATION

04U 001 33 51 3DS 68059 00903  
AIR FORCE WEAPONS LABORATORY/AFWL/  
KIRTLAND AIR FORCE BASE, NEW MEXICO 87117

ATT MISS MADELINE F. CANOVA, CHIEF TECHNI  
LIBRARY /WLIL/

POSTMASTER: If Undeliverable (Section 158  
Postal Manual) Do Not Return

*"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."*

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

## NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

**TECHNICAL REPORTS:** Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

**TECHNICAL NOTES:** Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

**TECHNICAL MEMORANDUMS:** Information receiving limited distribution because of preliminary data, security classification, or other reasons.

**CONTRACTOR REPORTS:** Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

**TECHNICAL TRANSLATIONS:** Information published in a foreign language considered to merit NASA distribution in English.

**SPECIAL PUBLICATIONS:** Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

**TECHNOLOGY UTILIZATION PUBLICATIONS:** Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

*Details on the availability of these publications may be obtained from:*

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION  
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546